

Opaque perfect lenses

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Abstract

The response of the “perfect lens”, consisting of a slab of lossless material of thickness d with $\varepsilon_s = \mu_s = -1$ at one frequency ω_0 is investigated. It is shown that as time progresses the lens becomes increasingly opaque to any physical TM line dipole source located at a distance $d_0 < d/2$ from the lens and which has been turned on at time $t = 0$. Here a physical source is defined as one which supplies a bounded amount of energy per unit time. In fact the lens cloaks the source so that it is not visible from behind the lens either. For sources which are turned on exponentially slowly there is an exact correspondence between the response of the perfect lens in the long time constant limit and the response of lossy lenses in the low loss limit. Contrary to the usual picture where the field intensity has a minimum at the front interface we find that the field diverges to infinity there in the long time constant limit.

Key words: Superresolution, Perfect lenses, Cloaking

1 Introduction

Recently there has been growing interest in superresolution, i.e. the fact that an image can be sharper than the wavelength of the radiation, which is in direct contrast to the proof of Abbe in 1873 that the resolution of a normal lens is at most about $\lambda/(2n)$ where λ is the wavelength and n is the refractive index. Initial progress towards superresolution had the characteristic feature

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that the decrease in spot size was accompanied by an unwanted sharp decrease in the intensity at the center of the spot relative to that of the Airy pattern (with an accompanying increase in sidelobe intensity): see [1] and references therein. Although its significance was not recognized at the time, a breakthrough came in 1994 when it was discovered that quasistatic line sources could have arbitrarily sharp images. Specifically it was found [2] that a coated cylinder with inner and outer radii r_c and r_s and having a real core dielectric constant ε_c , a shell dielectric constant ε_s close to -1 (with a small positive imaginary part) and a matrix dielectric constant $\varepsilon_m = 1$ would have some rather strange properties in the quasistatic limit (where the free-space wavelength is infinitely long compared to the structure). In particular a line source aligned with the cylinder axis and positioned outside the cylinder radius at a radius r_0 with $r_* < r_0 < r_*^2/r_s$ where $r_* = r_s^2/r_c$ would have an arbitrarily sharp image positioned at a radius r_*^2/r_0 outside the coated cylinder. This image would only be apparent beyond the radius r_*^2/r_0 ; closer to the coated cylinder the potential was numerically found to exhibit enormous oscillations. The reason that one finds an image at this radius is that it was shown that the effect of the shell was to magnify the core, so it was equivalent to a solid cylinder of radius r_* . By the method of images in two-dimensional electrostatics the field outside the equivalent solid cylinder is that due to the actual source plus an image source at the radius r_*^2/r_0 . However in contrast to electrostatics, the image source now lies in the physical region outside the coated cylinder. The paper [3] contains an in depth review of the results of the 1994 paper, correcting some minor errors.

In independent work, Pendry [4] claimed that line sources could have arbitrarily sharp images, even beyond the quasistatic regime, and he realized the deep significance of this result for imaging. His analysis suggested that the Veselago lens, consisting of a slab of material having thickness d , relative electric permittivity $\varepsilon_s = -1$, and relative magnetic permeability $\mu_s = -1$ (and thus having a refractive index $n = -1$) would act as a superlens perfectly imaging the fields near the lens and shifting them by the distance $2d$. Basically each interface of the lens acts like a mirror: Maxwell's equations are satisfied when the \mathbf{E} and \mathbf{H} fields on opposite sides of each interface are reflections of each other, and the two reflections about the two interfaces give a net translation of the fields by the distance $2d$. There were some flaws in Pendry's original analysis. In particular a point source at a distance $d_0 < d$ from the lens, could not have an actual point source as its image, since this would imply a singularity in the fields at the image point which cannot happen [5]. In fact there is no time harmonic solution in this case [6,7] since surface polaritons of vanishingly small wavelengths cause divergences [8]. While experiment has provided evidence for superresolution [9,10,11,12,13], to make theoretical sense of Pendry's claim one has to regularize the problem, say by making the slab lens slightly lossy or by switching on the source for a finite time. A careful analysis of the lossy case was made in [14,15], and a rigorous mathematical

proof of superlensing for quasistatic fields was given in [3] (see also [16] where a careful time harmonic analysis was given for real ε_s and μ_s close, but not equal to -1). Both for the quasistatic case [3] and for the full time harmonic Maxwell equations [17,18] it was shown that contrary to the conventional explanation where the field intensity has a minimum at the front interface of the lens, the field actually diverges to infinity in two anomalously resonant layers of width $2(d - d_0)$, one centered on the front interface and one centered on the back interface. Indications of large fields in front of the lens [16,19,20,21,22] were followed by definitive numerical evidence of enormous fields [23]. When $d_0 < d/2$ the resonant layers interfere with the source. It was discovered [17] (following a suggestion of Alexei Efros that the energy absorbed by the lens may be infinite), that finite energy point or line sources or polarizable point or line dipoles less than a distance $d/2$ from the lens become cloaked, and are essentially invisible from outside the distance $d/2$ from the lens. Thus the Vesalago lens, in the limit as the loss tends to zero does not perfectly image physical sources that lie closer to the lens than a distance $d/2$.

The following is a quick back of the envelope explanation of cloaking of a single polarizable dipole due to anomalous localized resonance (see the paper [17] for more details). First we should point that anomalous localized resonance is a phenomenon where as the loss in the system goes to zero the fields diverge to infinity within a specific region (the region of anomalous resonance) not associated with a change in character in that region of the underlying partial differential equation, but the fields approach smooth fields outside that region [3]. Now consider a polarizable (line or point) dipole outside the Vesalago superlens surrounded by fixed sources. Let E denote the electric field at the dipole in the absence of the dipole. Let E_r denote the field acting on the dipole, when its dipole moment is k and the surrounding fixed sources are absent (but the superlens is still present). Due to linearity one has that $E_r = c(\delta)k$, where δ is a parameter measuring the moduli governing the loss in the superlens with $\delta = 0$ corresponding to a loss-less lens, and $c(\delta)$ is in general tensorial but for simplicity of argument let us suppose that it is scalar valued. Due to anomalous resonance $|c(\delta)| \rightarrow \infty$ as $\delta \rightarrow 0$ when the dipole is within a distance $d/2$ from the superlens (and approaches zero otherwise). Now by superposition the total field acting on the dipole will be $E_t = E + E_r$, and the induced dipole moment will be $k = \alpha E_t$ where α is the polarizability of the dipole. So we have that $k = \alpha[E + c(\delta)k]$ and solving for k gives $k = \alpha_* E$ where the “effective polarizability”

$$\alpha_* = \frac{\alpha}{1 - \alpha c(\delta)} \quad (1.1)$$

goes to zero (and is asymptotically almost independent of α) when the dipole is positioned within the cloaking region $0 < d_0 < d/2$ where $|c(\delta)| \rightarrow \infty$. Basically the polarizable dipole causes the superlens to build up a localized

resonance and the fields from this resonance almost exactly cancel the field that would otherwise act on the polarizable line dipole: in effect, the polarizable line dipole feels a vanishingly small field. So its induced dipole moment is close to zero and consequently it perturbs the field only slightly outside the resonant region. It is effectively invisible. This simple argument has to be replaced by more sophisticated (energy based) arguments to establish the cloaking of collections of dipoles. In contrast to superlensing, which requires very low loss materials to get a significant enhancement of resolution [15,18] cloaking effects occur for materials which have moderate loss: see figure 3 in [17].

The hope has persisted that a source turned on at time $t = 0$ would be perfectly imaged by a lossless Veselago lens (the perfect lens) as $t \rightarrow \infty$. This was first suggested by Gómez-Santos[24] and subsequently Yaghjian and Hansen[18] gave a detailed analysis. Both papers took into account the fact that due to dispersion $\mu_s(\omega)$ and $\varepsilon_s(\omega)$ can only equal -1 at one frequency ω_0 . At nearby frequencies one has

$$\begin{aligned}\varepsilon_s(\omega) &= -1 + a_\varepsilon(\omega - \omega_0) + O[(\omega - \omega_0)^2], \\ \mu_s(\omega) &= -1 + a_\mu(\omega - \omega_0) + O[(\omega - \omega_0)^2],\end{aligned}\tag{1.2}$$

where, due to causality, the dispersion coefficients (with $\varepsilon_s(\omega_0) = \mu_s(\omega_0) = -1$) necessarily satisfy the inequalities [25,18]

$$a_\varepsilon = \left. \frac{d\varepsilon_s}{d\omega} \right|_{\omega=\omega_0} \geq \frac{4}{\omega_0}, \quad a_\mu = \left. \frac{d\mu_s}{d\omega} \right|_{\omega=\omega_0} \geq \frac{4}{\omega_0},\tag{1.3}$$

which force them to be positive. [These inequalities are also a corollary of bounds derived in [26]. To see this, suppose ω and ω_0 belong to a frequency interval where ε_s is real, and that we seek bounds which correlate the values that $\varepsilon_s(\omega)$ and $\varepsilon_s(\omega_0)$ can take. Without loss of generality let us suppose that $\omega^2 > \omega_0^2$. Then from equation (6) in [26] we have the sharp bound

$$\varepsilon_s(\omega) - 1 \geq \max\{\varepsilon_s(\omega_0) - 1, \omega_0^2[\varepsilon_s(\omega_0) - 1]/\omega^2\}\tag{1.4}$$

which when $\varepsilon_s(\omega_0) = -1$ reduces to

$$\varepsilon_s(\omega) \geq 1 - 2\omega_0^2/\omega^2.\tag{1.5}$$

By substituting the Taylor expansion (1.2) in this inequality and letting $\omega \rightarrow \omega_0$ we obtain the first inequality in (1.3). The second inequality is obtained by similar arguments applied to the magnetic permeability.]

For simplicity it is assumed that the surrounding matrix material has $\mu_m = \varepsilon_m = 1$ for all frequencies. It was shown in the papers [18,24] that the field at any given time would be finite except at the source. Also figure 1 in [24] shows the field has a local intensity minimum at the front interface and it was claimed in [18] that as $t \rightarrow \infty$ the field would diverge only in a single layer of width $2(d - d_0)$, centered on the back interface. However, here we will show that, again contrary to the conventional picture, the situation is precisely analogous to what occurs in a lossy lens as the loss goes to zero. The field also diverges to infinity in the layer of width $2(d - d_0)$ centered on the front interface, and as a consequence cloaking occurs when the source is less than a distance $d/2$ from the lens. The image of a constant energy source in this cloaking region becomes rapidly dimmer and dimmer as time increases. So instead of the lens being perfect, it is actually opaque to such sources, and cloaks them: not only is the source dim behind the lens, it is also dim in front of the lens. Essentially all of the energy produced by the source gets funneled into the resonant regions which continually build up in intensity. Thus the claim [24] that “*even within the self-imposed idealizations of a lossless (for $\omega = \omega_0$) and purely homogeneous, left handed material, Pendry’s perfect lens proposal is correct*” has to be qualified. It is *only true* for physical sources located further than a distance $d/2$ from the lens. For physical sources located less than a distance $d/2$ from the lens the image is completely different from what would appear if the lens were absent because the source interacts with the resonant fields in front of the lens. Although our analysis assumes a point source, any source of finite extent can be viewed as a superposition of dipolar sources and will create fields in front of the lens that interact with the source.

2 Analysis

Simple energy considerations indicate that something strange must happen when $d_0 < d/2$. From equation (62) in [18] we see that a source of constant strength E_0 switched on at $t = 0$ creates an electric field which near the back interface (and outside the lens) scales approximately as

$$E \sim E_0 t^{1-d_0/d}. \quad (2.1)$$

The stored electrical energy $S_E(t)$ will scale as the square of this, and consequently the time derivative of the stored electrical energy will scale approximately as

$$\frac{dS_E}{dt} \sim E_0^2 t^{1-2d_0/d}, \quad (2.2)$$

which blows up to infinity as $t \rightarrow \infty$. If the source produces a bounded amount of energy per unit time we have a contradiction. The conclusion is that if the energy production rate of the source is bounded then necessarily E_0 must decrease to zero as $t \rightarrow \infty$. (If it approached any other equilibrium value then again we would have a contradiction). This sounds rather paradoxical but it could be explained if there was a resonant region in front of the lens, creating a sort of optical molasses, requiring ever increasing amounts of work to maintain the constant strength E_0 .

Let us see that there is a resonant region in front of the lens through an adiabatic treatment of the problem. For simplicity we assume a TM line dipole source located along the Z -axis (which we capitalize to avoid confusion with $z = x + iy$) and that the slab faces are located at the planes $x = d_0$ and $x = d_0 + d$. Instead of assuming that the source is turned on sharply at $t = 0$ and thereafter remains constant we assume that it has been turned on exponentially slowly beginning in the infinite past. The source generates a field with the plane wave expansion

$$H_Z^{\text{dip}}(x, y, t) = \int_{-\infty}^{\infty} dk_y a(k_y) e^{i(k_x x + k_y y - \omega t)} \quad \text{with} \quad k_x = \sqrt{\omega^2/c^2 - k_y^2}, \quad (2.3)$$

for $x > 0$, which interacts with the lens, where the coefficients $a(k_y)$ need to be determined and the square root in (2.3) is chosen so $\text{Im } k_x > 0$ to ensure that the waves due to the source decay as x increases. The frequency

$$\omega = \omega_0 + i/T \quad (2.4)$$

is complex and T is a measure of the time the source has been “switched on” until time $t = 0$. It does not make sense to analyse this model in the limit as $t \rightarrow \infty$ since everything diverges exponentially in that limit. Rather we consider the model at time $t = 0$ at which point the source has been approximately constant for a very long period of time of the order of T . Thus investigating the asymptotic behavior as $T \rightarrow \infty$ at $t = 0$ in this model is analogous to investigating the asymptotic behavior as $t \rightarrow \infty$ of a constant amplitude source which has been switched on at time $t = 0$.

For a dipole line source we have

$$H_Z^{\text{dip}}(x, y, t) = \frac{\pi \omega_0 e^{-i\omega t}}{2} \left(-k^o \frac{\partial}{\partial x} + i k^e \frac{\partial}{\partial y} \right) H_0^{(1)} \left((\omega/c) \sqrt{x^2 + y^2} \right), \quad (2.5)$$

in which $H_0^{(1)}$ is a Hankel function of the first kind and k^e is the (possibly complex) strength at $t = 0$ of the dipole component which has an associated

electric field with even symmetry about the x axis and k° is the (possibly complex) strength at $t = 0$ of the dipole component which has an associated electric field with odd symmetry about the x axis: these dipole strengths have been normalized to agree with the definitions in [3] and [17]. By substituting the plane wave expansion [see formula (2.2.11) in [27]]

$$H_0^{(1)}\left((\omega/c)\sqrt{x^2+y^2}\right) = \frac{1}{\pi} \int_{-\infty}^{\infty} dk_y \frac{e^{i(k_x x + k_y y)}}{k_x}, \quad (2.6)$$

with

$$k_x = \sqrt{\omega^2/c^2 - k_y^2}, \quad (2.7)$$

in (2.6) we see that

$$a(k_y) = -\omega_0[k^e(k_y/k_x) + ik^\circ]/2. \quad (2.8)$$

We look for a particular solution of Maxwell's equations where all the fields, and not only the source, vary with time as $e^{-i\omega t}$ where ω is given by (2.4). This solution is obtained by substituting this complex value of ω into the time harmonic Maxwell's equations. Specifically with $\omega = \omega_0 + i/T$ and with the lens having the least possible dispersion, ε_s and μ_s will according to (1.2) have the complex values

$$\varepsilon_s = -1 + ia_\varepsilon/T + O(1/T^2), \quad \mu_s = -1 + ia_\mu/T + O(1/T^2), \quad (2.9)$$

In other words, apart from the modulating factor of $e^{-i\omega t}$, the mathematical solution for the fields is exactly the same as for a lossy material with μ_s'' and ε_s'' approximately proportional to $1/T$ for large T . A correspondence of this sort was noted before [18] but not fully exploited. By this argument it immediately follows that for fixed k^e and k° the fields will diverge as $T \rightarrow \infty$ in *two* possibly overlapping layers of the same width $2(d - d_0)$ one centered on the back interface and one centered on the front interface. In particular, in front of the lens, with $2d_0 - d < x < d_0$, equations (4.18) and (4.19) of [17] imply

$$H_Z(x, y, t) \approx H_Z^{\text{dip}}(x, y, t) - \omega_0 e^{-i\omega t} \{[g^e(z) - g^e(\bar{z})]/2 + [g^\circ(z) + g^\circ(\bar{z})]/(2i)\}, \quad (2.10)$$

where $z = x + iy$, $\bar{z} = x - iy$ and

$$g^p(z) = -iqk^p[a_\varepsilon/(2T)]^{(2d_0-d-z)/d} Q_0(2d - 2d_0 + z), \quad (2.11)$$

with

$$Q_0(b) = \frac{\pi}{2d \sin[\pi b/(2d)]}, \quad (2.12)$$

in which $q = 1$ for $p=e$ and $q = -1$ for $p=o$. Thus we see that $g^p(z)$ and hence $H_Z(x, y, t)$ diverges as $T \rightarrow \infty$ within a distance $d - d_0$ from the front of the lens. When $d_0 < d/2$ this resonant region interacts with the source creating the “optical molasses” that we mentioned. We have not done the computation, but presumably if one took $k^o = 0$ and chose k^e to depend on T in such a way that the source produces a given (T independent) amount of energy at time $t = 0$ then one would find as $T \rightarrow \infty$ that the field would be localized and resonant in two layers of width d which touch at the slab center. We remark that such field localization was found in the quasistatic case in the low loss limit [17] and also when two opposing sources are placed a distance $d/2$ behind and in front of the lens [28,29]

We only considered a particular solution to the equations. The general solution is of course the sum of a particular solution plus a solution to the homogeneous equations with no sources present, which we call a resonant solution. Since the lens is lossless, energy must be conserved and so a resonant solution which is zero and has zero total energy in the infinite past, must be zero for all time. Therefore the particular solution we considered is the only solution which satisfies the boundary condition of being zero in the infinite past.

No immediately apparent problems occur for line sources with d_0 between $d/2$ and d . While the stored electrical energy $S_E(t)$ in the resonant regions increases without bound, we see from (2.2) that the rate of increase diminishes with time. Similarly the rate of increase of magnetic energy diminishes with time. Therefore the image of such sources will get brighter and brighter as $t \rightarrow \infty$ approaching the same brightness as the original source without the lens present. However because the energy stored in the resonant regions is so large it may be the case that slight variations in the intensity of the source or slight non-linearities or slight inhomogeneities in the permeability and permittivity of the lens will scatter radiation and destroy the “perfect image”. The spatial dispersion of the dielectric response of the slab will also limit resolution [30]. Finally we remark that we have assumed that the radiation coming from the source is coherent.

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